BOUNDARY of an ELLIPSE

A 10-minute mathematics zine for #mathober 2026

by @stecks

Ellipses are conic sections:

So if you hang a loop of string on a cone, it will form the boundary of an ellipse!

An ELLIPSE is a shape that's similar to a circle:

But instead of having the same width theight, it's longer in one direction:

An ellipse has two FOCI (singular: focus) which lie inside the shape

The equation for the ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

where a is half the height of the ellipse and b is half the width.

The boundary of the ellipse is also the set of points where the sum of the distances to the two foci is equal to $2a$ (the width of the ellipse).

This means you can draw an ellipse by sticking two pins into a page at the foci, and putting a loop of string around both foci and pen, and moving it around.

The area, meanwhile, is simply given by

$$A=\pi ab$$

The length of the boundary of an ellipse cannot be written as a simple formula in terms of a & b

It can be calculated as:

$$P=4a\int_0^{\frac{\pi}{2}}\sqrt{1-e^2\sin^2\theta}\,d\theta$$

where

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

the eccentricity of the ellipse

ZEBRA GRAPH

A 10-minute maths zine for

#mathober 2026

by @stecks

On a 4-by-4 chess board, these are the moves a zebra piece can make.

We can think of this as a graph, and these are called ZEBRA GRAPHS.

The 5-by-5 board has two separate closed loops (+1 unreachable square)

As we increase the size of the board, the graph has more fun properties.

From 6-by-6 onwards, the graph is connected into one piece. for a 10-by-10 board, there's a Hamiltonian path!

All zebra graphs are two-colourable

(which can be seen by remembering it's on a chessboard, and each move takes it between a black + white square!)

A ZEBRA is a piece which moves across three and up two spaces.

(Nobody uses them for playing chess, but we can play with them mathematically!)

A KNIGHT is a chess piece that moves in an L-shape (across one, up two)

But people have defined other kinds of made-up chess pieces too, that move in similar ways.

Other fun animal pieces

Antelope: →4 ↑3

Camel: →1 ↑3

Giraffe: →1 ↑4

(implying the existence of a giraffe graph!)

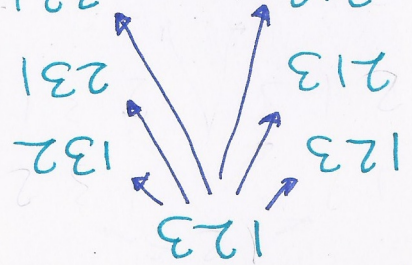
PERMU-TATION

A 10-minute maths zine for #mathober 2026

by @stecks

A permutation is a reordering of a finite set of objects.

e.g for a set of 3 objects, there are 6 possible permutations:

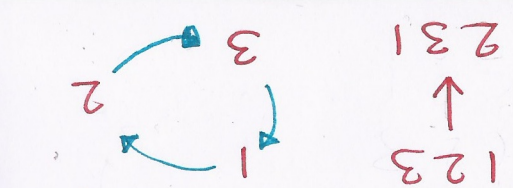


There are many types of permutation you can define.

KNOW YOUR PERMUTATIONS

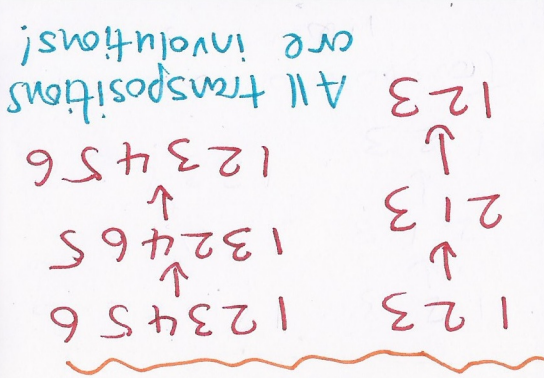
Cyclic permutation

One where the objects move around in a cycle.



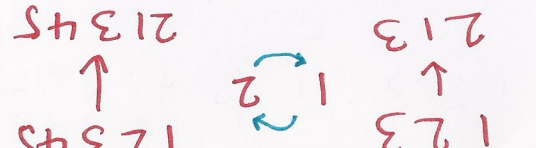
Involution ★

A permutation which, if you apply it twice, everything goes back to where it was.

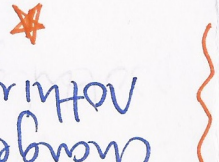


Transposition ★

A permutation in which two objects swap place.



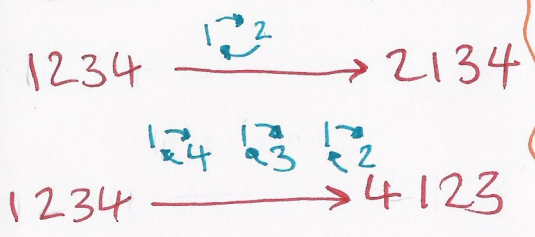
Identity permutation (Trivial permutation)



All permutations can be created by combining transpositions. The number of transpositions needed allows us to categorise them all as:

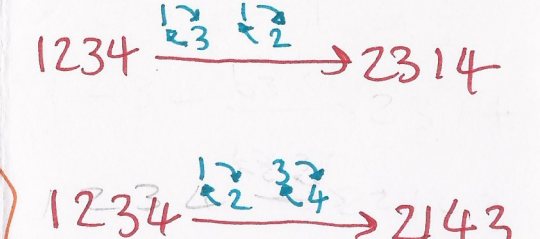
Odd permutations

Ones made from an ODD number of transpositions



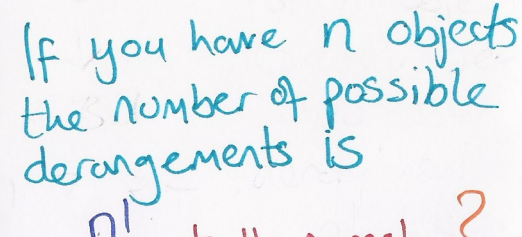
Even permutations

Ones made from an EVEN number of transpositions



Derangement ?

A permutation where every object ends up not where it started.



If you have n objects the number of possible derangements is

$$\frac{n!}{e}$$

to the nearest integer ?

LINEAR RECURRENCE

A 10-minute maths zine for #Mathober 2026

by @steeles

A linear recurrence is a sequence made by combining previous terms in the sequence in a linear way.

The most famous example is the FIBONACCI SEQUENCE

This is defined by $a_{n+1} = a_n + a_{n-1}$

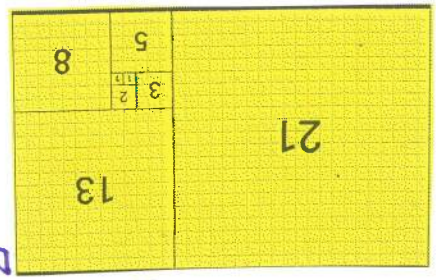
(Each term is the sum of the one before it and the one two before)

Conventionally, we set $a_0 = a_1 = 1$

The sequence goes

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89... (OEIS A000045)

The Numbers can be visualised as the side lengths of squares that fit together into a nice rectangle!



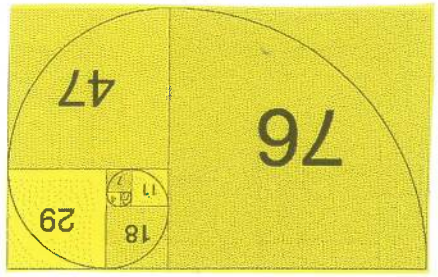
But fibonacci is not the only cool linear recurrence!

If instead we start with $a_0 = 2, a_1 = 1$ we get

The LUCAS NUMBERS which go

2, 1, 3, 4, 7, 11, 18, 29... (A000032 OEIS)

Their rectangle looks like this:



We can define other kinds of linear recurrence e.g.

$Q_n = Q_{n-2} + Q_{n-3}$

(with $Q_0 = Q_1 = Q_2 = 1$)

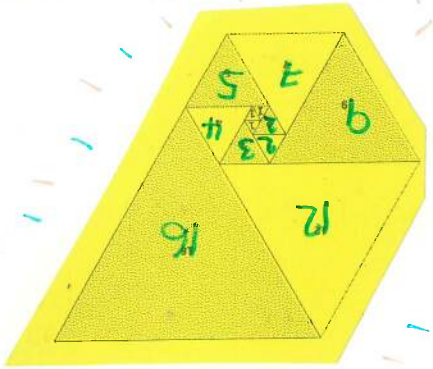
gives the PADOVAN SEQUENCE

This goes

1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, ... (A000931 in OEIS)

The triangles increase in size as they spiral outwards from the middle.

This can be visualised as this cool shape made from triangles:



The same diagram can also be used to prove that in the Padovan sequence,

$a_n = a_{n-1} + a_{n-5}$

(eg $16 = 9 + 7$ but also $16 = 12 + 4$, as the edges line up along the edge of the larger triangle!)



INELLIPSE

A 10-Minute Maths Zine for #Mathober 2026

by @stecks

You may already be familiar with circles you can define relative to a triangle:

But you can also use a triangle to define other conic sections! These are brilliantly called

INCONICS

absolutely iconic

NO GOOGLE! DON'T CORRECT ME

and

CIRCUMCONICS

An INELLIPSE is the ellipse version of this (other conics are available)

For a given triangle △ there are many ways to define an inellipse.

In fact, every point in the medial triangle has an inellipse you can define with that point as its center! (ii)

The LARGEST inellipse for any triangle is the STEINER INELLIPSE

This is the ellipse that lies tangent to the sides at their midpoints.

You can also define the STEINER CIRCUMELLIPSE

The area of the Steiner inellipse is 1/4 of the area of the Steiner circumellipse!

If the triangle is equilateral, the Steiner inellipse + Steiner circumellipse are just the inCircle and circumCircle!

You can also define inellipses for a quadrilateral:

All the centres of inellipses of a quadrilateral lie on the line connecting the midpoints of its two diagonals!

CONNECTED TOPOLOGY

A 10-minute maths zine for #mathober 2026

by @steeks

In topology, we can define a space as CONNECTED if it cannot be split into two seperate (open) subspaces.

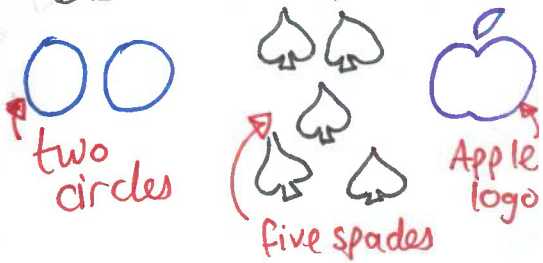
Roughly, this means it is made from one single piece.



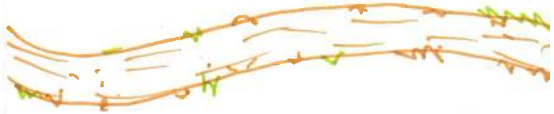
Connected



Disconnected



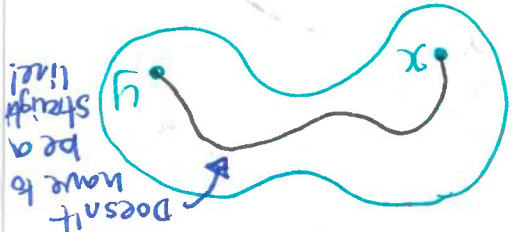
For a stronger condition, we can require a space to be PATH CONNECTED.



This means any two points in the space can be connected by a continuous path.



We describe this path as the image of the interval $[0,1]$ mapped into the space, with $f(0) = x$, $f(1) = y$ and the function has to be continuous.



All spaces which are path connected are also connected, but there are some weird counterexamples of spaces which are connected but not path connected.

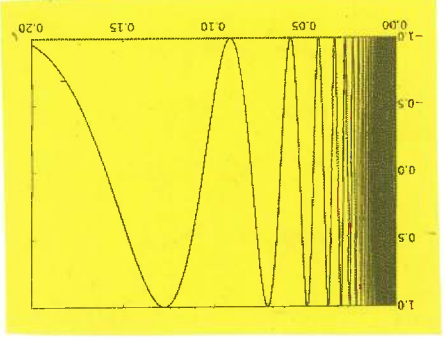
The TOPOLOGIST'S SINE CURVE is one such example.

It is the set

$$T = \{(x, \sin(\tfrac{1}{x})) : x \in (0,1]\} \cup \{(0,0)\}$$

That is, the image of $(0,1]$ (not including 0) under $\sin\frac{1}{x}$, plus the point $(0,0)$.

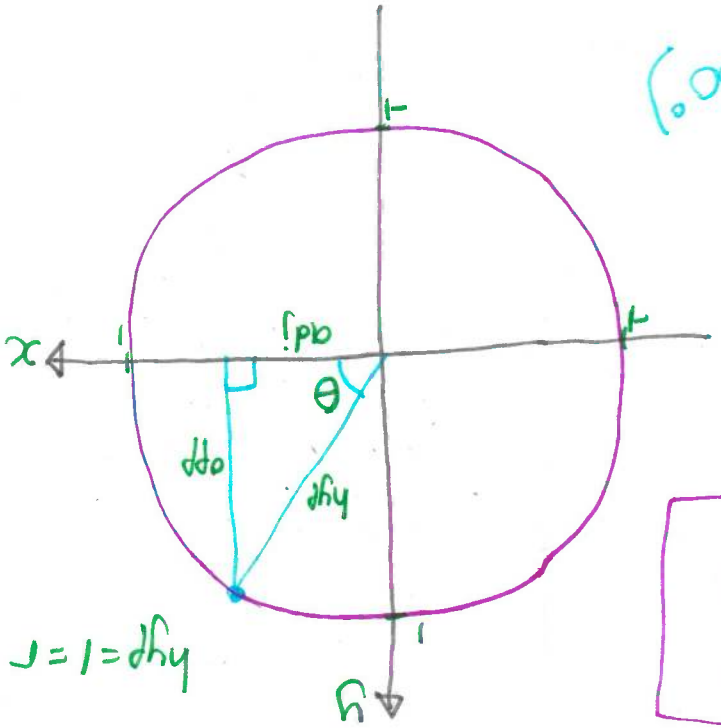
It is connected!



However, T is not path connected. Any path from a point on the curve to $(0,0)$ would need to be infinitely long, so the map can't be defined

The oscillations of the sine curve keep getting closer together.

Textbook textbook example!



hyp = 1 = r

$\Rightarrow \theta = \frac{\pi}{3}\ (60°)$

$y = \frac{\sqrt{3}}{2}$

$x = \frac{1}{2}$

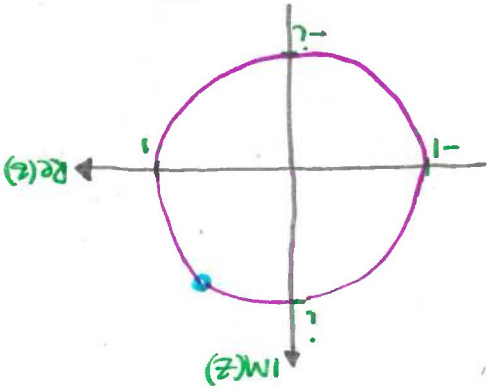
e.g.

3. $x = \cos\theta$, $y = \sin\theta$

e.g. $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

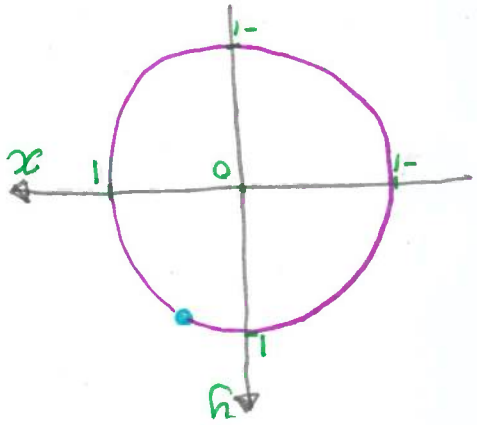
$z = a + bi \in \mathbb{C}$

2. $z = e^{i\theta}$



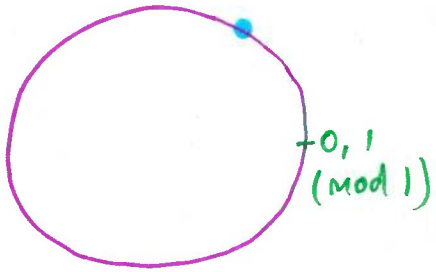
e.g. $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$

1. $x^2 + y^2 = 1$

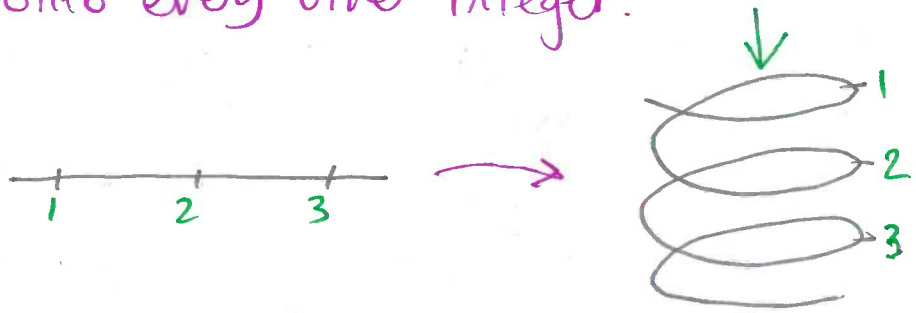


4. $\mathbb{T} = \mathbb{R}/\mathbb{Z}$

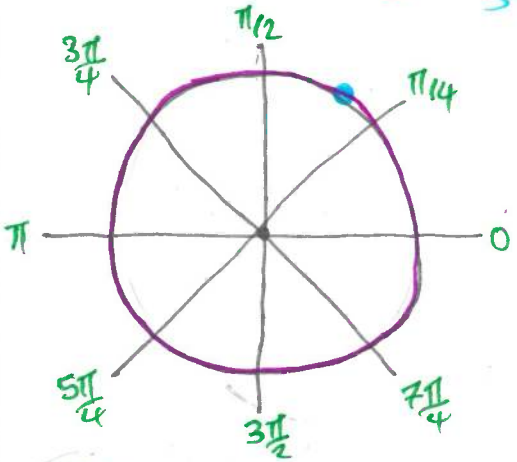
(quotient group) e.g. $\frac{1}{6}$



Take the real line and stick every integer onto every other integer.



5. $r = 1$ e.g. $\theta = \frac{\pi}{3}$





(polar coordinates)



FIVE CIRCLES

A 10-minute maths zine for #Mathober 2026

by @stecks

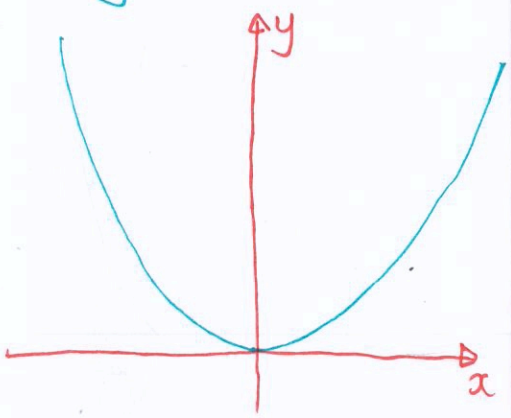
PARABOLIC ROTATION

10-minute AMaths zine for #Mathober 2026

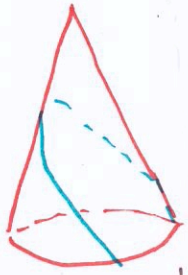
by @stecks

A parabola is the name given to a curve of the form

$y = x^2$



It's a conic section: If you slice the cone at an angle you get a parabola.



I was recently intrigued to find (on Wolfram Ma[illegible],

In the geometry → transformations → miscellaneous transformations → Parabolic Rotation section, this definition of a map (with no context, or indication of where it came from).

The map

$$x' = x + 1 \quad (1)$$

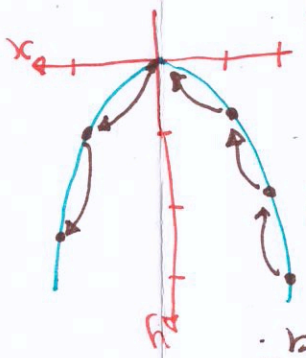
$$y' = 2x + y + 1, \quad (2)$$

which leaves the parabola

$$x'^2 - y' = (x+1)^2 - (2x + y + 1) = x^2 - y \quad (3)$$

invariant.

x, y	f(x), f(y)
0, 0	1, 1
1, 1	2, 4
-1, 1	0, 0
-2, 4	-1, 1



The map is lovely! It takes each point on the parabola and moves its x-value up by 1, while keeping it on the parabola.

I love made a Geogebra file that implements it so you can play:

Kahestechtes.co.uk/mob-ggb



SEE ALSO

Parabola, Rotation

K-CYCLIC GRAPHS

A 10-minute Maths zine for #Mathober 2026

by @stecks

A K-cyclic graph is one on which you can trace out a closed path along K edges, which ends where it started.

There are 12 7-cyclic graphs. They are* called:

When studying a function like

$$f(x) = \frac{4x+6}{x}$$

mathematicians are often interested in finding out two things:

- where the function is defined (has a meaningful value)
- where it equals zero

for example, if $x = -1.5$, then

$$\frac{4x+6}{x} = \frac{4(-1.5)+6}{-1.5} = 0$$

We say -1.5 is a zero of this function.

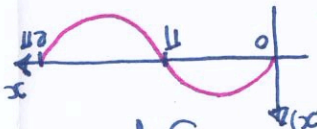
But if $x = 0$, we can't say what $\frac{4x+6}{x}$ is.

So the function is defined for all values of x except 0.

A function can have multiple zeros, or be undefined at many points

e.g $f(x) = \sin x$

$f(x) = 0 \Leftrightarrow x = 0, \pi, 2\pi, 3\pi, \ldots$



but $f(x) = \tan x \left(= \frac{\sin x}{\cos x}\right)$ is undefined when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

(because this triangle $\leftarrow$ is impossible)



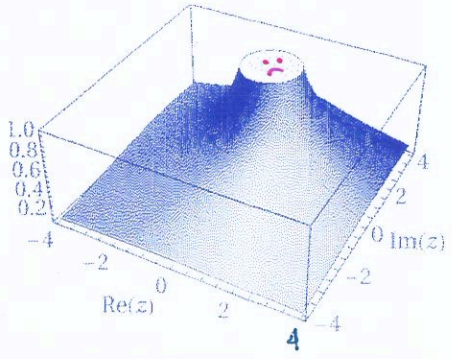
A point at which a complex function is not defined is called a pole.

These occur when we have zs on the bottom of a fraction:

e.g $f(z) = \frac{1}{z-2i}$

has a pole at $z = 2i$

(I cheated and made a plot on Wolfram|Alpha)

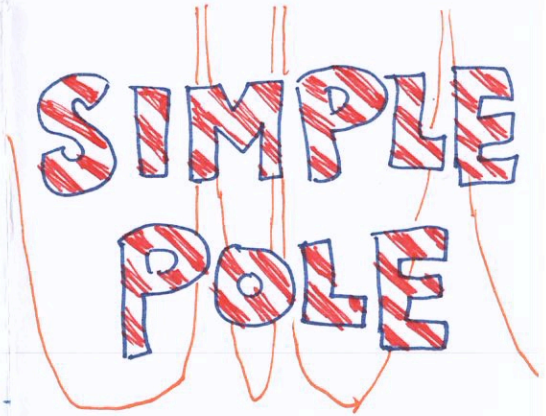


Approaching this point from any direction in the complex plane will see the value of the function become large.

The order of a pole tells us what power of z is on the bottom of the fraction.

e.g $f(z) = \frac{1}{z^2}$ has a pole of order 2 at $z=0$.

A SIMPLE POLE is a pole of order 1. If $f(z)$ has a simple pole at z_0, $f(z)\cdot(z-z_0)$ does not!



A 10-Minute Maths zine for # Mathober 2026

by @stecks

We can also define these things for COMPLEX FUNCTIONS:

$f(z) : \mathbb{C} \to \mathbb{C}$

(they are harder to draw plots of!)

A point at which a complex function equals 0 is still called a zero.

e.g $f(z) = z - 1$ has a zero at $z = 1$.

THE PROBLEM OF POINTS

A 10-minute Maths zine for #Mathober 2026

by @stecks

Imagine two players are playing a game. There are multiple rounds in the game, and each player has an equal chance of winning each round.

e.g COINFLIPS 

or a skill game where they are equally good.

They each stake some money 💰, and decide to play until one player has won n rounds, at which point that player wins all the money.

HOWEVER!

After K rounds, the game has to be cancelled.

How can they fairly divide ☺ the prize money? ÷

One solution could be that each player receives a proportion of the money corresponding to how many rounds they have won so far.

e.g Player 1 | 3 : 2 | Player 2

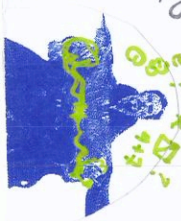
Player 1 gets £60 ← from £100 pot, Player 2 gets £40

This works, but can be unfair - if e.g $K=1$ and one player wins the whole pot!

A better option might be to base it on the difference in score between the players and the total number of rounds.

But this gives the same split to player scoring 9:8/10 as it does to players on 3:2/10. Is it fair?

The solution widely considered to be the best involves studying all the possible futures the game could have, from the point it stops.



If player 1 needs r more rounds to win, and player 2 needs s, the game might need $r+s-1$ more rounds to decide it (in the worst case)

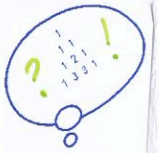
We could list all the 2^{r+s-1} possible outcomes (including the ones where someone wins before we get to $r+s-1$ rounds) and check who would win in each case - then split the money that way. This is given by:

$$\sum_{k=0}^{s-1} \binom{r+s-1}{k} \cdot \frac{1}{2^{r+s-1}} \sum_{k=s}^{r+s-1} \binom{r+s-1}{k}$$

"choose"

Many mathematicians have studied this problem, coming up with different solutions.

Blaise Pascal worked on this last method, and it inspired him in his work on what's now known as Pascal's triangle, and the notion of expected value!





RECIPROCAL PRIME CONSTANT

A 10-minute maths zine for #Mathober 2026 by @stecks

The harmonic series is a series made by summing the values of $\frac{1}{x}$ for $x = 1$ up to n

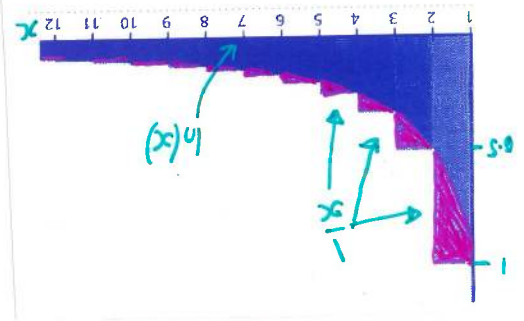
If you go

$1, \quad 1+\frac{1}{2}, \quad 1+\frac{1}{2}+\frac{1}{3}, \quad$ etc

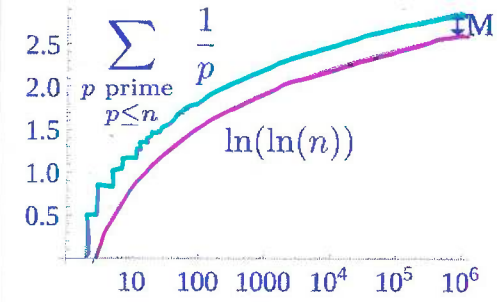
$1, \quad 1.5, \quad 1.8\dot{3}, \quad 2.08\dot{3}$

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}, \quad$ etc

In the limit, the area of the purple region converges to a fixed value.



If you plot this on the same axes as a plot of $\Delta(n)$, it looks like this:



But it grows at roughly the same rate as $\ln(\ln(n))$

This time, instead of the difference increasing having a limit, the two curves have a limiting difference: for large n they are a fixed distance apart.

$$M = \lim_{n\to\infty}\left(\sum_{\substack{p \text{ prime} \\ p\le n}} \frac{1}{p} - \ln(\ln(n))\right)$$

is called the Meissel-Mertens constant, and

$M \approx 0.261497\ldots$

(OEIS A077761).

There's also a fun connection between the two constants:

$$M = \gamma + \sum_{p \text{ prime}} \left[\ln\left(1-\frac{1}{p}\right) + \frac{1}{p}\right]$$

Euler-Mascheroni

Meissel-Mertens constant

aka Reciprocal Prime Constant!

It is $\gamma = 0.5772\ldots$ and is called the Euler-Mascheroni constant.

Even though the Harmonic Series is divergent, Euler observed that it grows at roughly the same rate as $\Delta(x)$ so their difference is finite.

This series also diverges

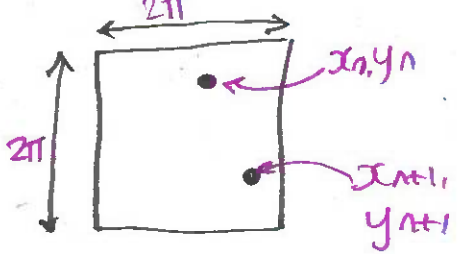
$$\sum_{\substack{p \text{ prime} \\ p\le n}} \frac{1}{p}$$

Turns out, we can do a similar trick with the series made by summing the reciprocals of just the primes.

STANDARD MAP

A 10-minute Maths zine for #mathsbe 2026 by @stetics

The standard map (aka the Chirikov map) takes points in a square and maps them onto other points in the same square:



It is defined as

$$x_{n+1} = x_n + K\sin(y_n)$$
$$y_{n+1} = y_n + x_{n+1}$$

It can be used to study a dynamical system where x and y are the momentum and angular position of a rotating stick, which behaves chaotically for some K.

But we can also use it to map some standards!

Here it is applied to the standard of the Royal Family of the Netherlands.

(Full disclosure: this was a more than 10-min zine if you include the time I spent debating Python)





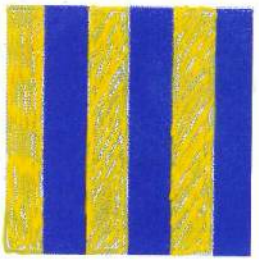
And here it is applied to the Ukraine, a flag representing the native peoples of the Andes (including Bolivia, Peru, Chile, Argentina, Ecuador & Southern Colombia):





Here's the effect of applying the map ($K=5$) to the Maritime signal flag for the letter G:

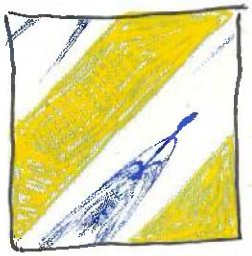
(BTW 'standard' is another word for a flag, sorry not sorry)

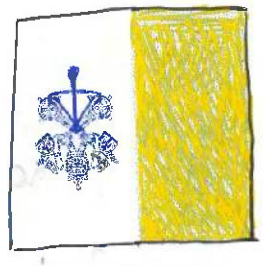




Here's the image under the standard map of the flag of Vatican City, one of only two square flags of countries in the world.

(yes, this is just flags now for the rest of the zine)





Here it is on the flag of Switzerland (one of only two square flags of countries in the world).





ICOSI-TETRAGON

A 10-minute maths zine for #mathober 2026

by @stecks

Polygons are often named after their number of sides:

- trigon (occasionally also called 'triangle')
- tetragon (nobody ever calls this anything else)
- pentagon
- hexagon (bestagon)
- heptagon
- STOP listing polygons!

An ICOSITETRAGON has 24 sides!

Here are some examples of icositetragons illustrated using irregular icositetragons.

QUARE CITATIONS

IT COAST REGION

COOING ARTISTE

GOAT IS NOT ERIC

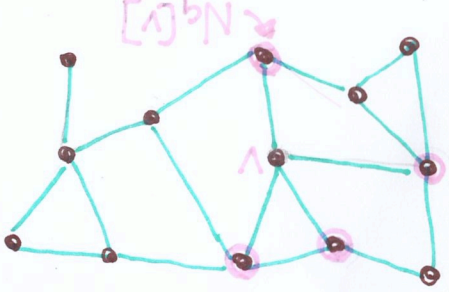
CORING TOASTIE

IRREDUNDANT

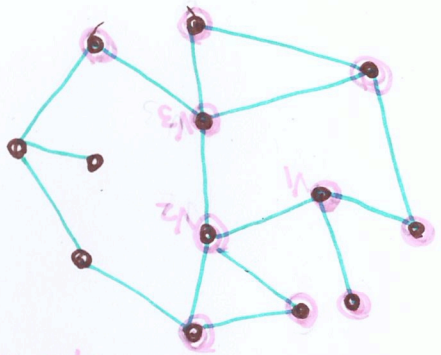
A 10-minute maths zine for #Mathober 2026

by @stecks

In a graph, we define the neighbourhood of a vertex v to be all the vertices connected by an edge to v.



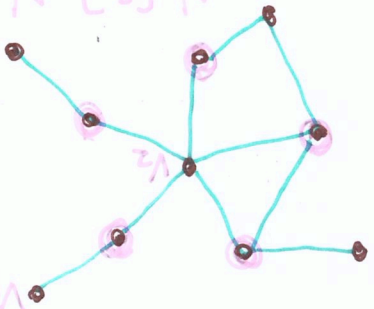
We can also define a neighbourhood for a set of vertices

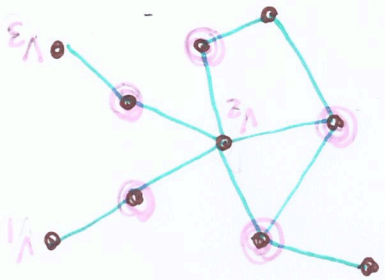


If removing a vertex from the set would not change the neighbourhood of that vertex set, we can think of that vertex as being redundant

$N_q[S-\{v\}] = N_q[S]$

$\Rightarrow v$ is redundant



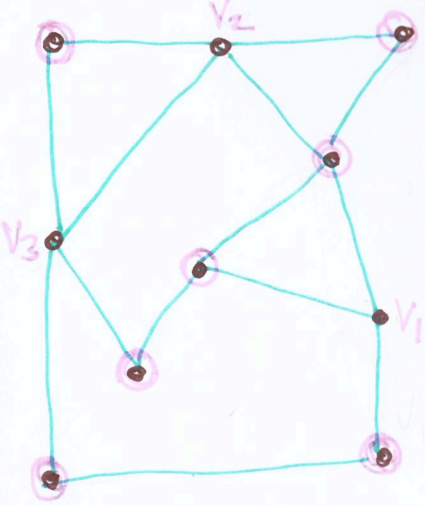


A vertex set S is IRREDUNDANT if none of its vertices are redundant - removing any of them would change the neighbourhood of the vertex set.

$N_q[S-\{v\}] \neq N_q[S]$

$\forall v \in S$

Another way of thinking about this is that for each vertex in an irredundant set, there is an adjacent vertex that is not adjacent to any other vertex in the set.





A ROULETTE is a curve made by tracing the motion of a point on a curve as it moves around another curve (without slipping). For example, if we roll a circle around the outside of another circle, and trace the path of one point on the outside circle, we get an epicycloid.

If we take a parabola and roll it around another parabola, tracing its centre point gives a roulette known as a **cissoid of diocles**.

ISOCHRONOUS CURVE

A 10-Minute Maths zine for #Mathober 2026

by @stecks

This is a curve for which the time taken for a mass (without friction) to roll down from any point on the curve to the bottom is the same!

One of the most well known roulette curves is the Cycloid.

This is created by rolling a circle along a straight line, and tracing out the path taken by a single point on the edge of the circle.

Cycloids have many uses in engineering and lots of cool properties. In particular, a cycloid is an isochronous curve.

The dimension invariance theorem says that

$$\mathbb{R}^n \simeq \mathbb{R}^m \Leftrightarrow n = m$$

There is no bicontinuous map between Euclidean spaces of different dimension. Even though they all have the same number of points in them (infinitely many) we cannot define maps between them which are continuous in both directions.

DIMENSION INVARIANCE

$\simeq$ $\simeq$ $\simeq$ $\simeq$

A 10-minute maths zine for #Mathober 2026

by @SteCKS

In topology, we say two spaces are homeomorphic if we can define a continuous bijection between them (whose inverse is also continuous). This means a map taking points from one space to the other, which is:

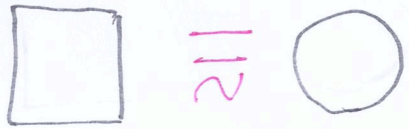
- one-to-one
(each point has a unique image under the map)
- onto
(every point is mapped to by something)

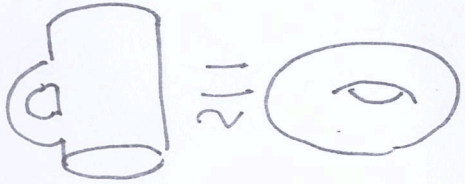
(Together these also mean the map is invertible and can be used both ways)

- continuous
(points which are close together get mapped to points which are still close together).

Roughly speaking, this means I can take one object and deform it into the other in a continuous way.

e.g. a circle is homeomorphic to a square





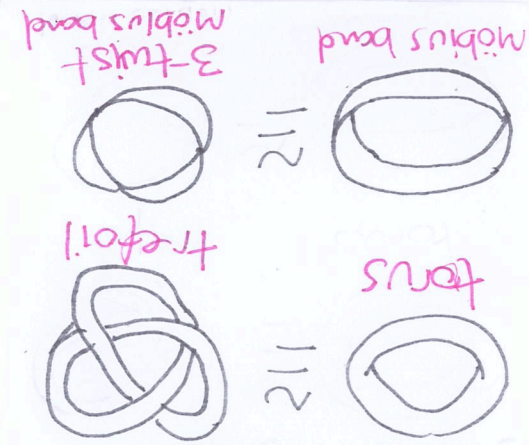
famously, a donut is homeomorphic to a teacup!

Other examples:

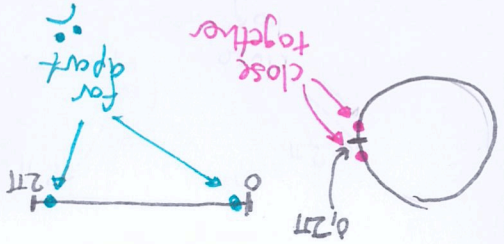
$(a,b) \simeq \mathbb{R}$

open interval → infinite real line

by $f(x) = \frac{1}{a-x} + \frac{1}{b-x}$



The deformation taking one space to another can take place in higher dimensions. So, exciting!



The map $f:[0,2\pi) \to S^1$ (from a half-open interval to the unit circle) given by $f(\varphi) = (\cos\varphi, \sin\varphi)$ is not a homeomorphism, as its inverse is not continuous at 0.

BIFLECNODAL POINT

A 10-minute Maths zine for #Mathober 2026

by @stecks

An **inflection point** of a curve is a point on the curve where the curvature changes sign - from curving one way, it becomes straight like briefly, then curves the other way.

If a curve is the graph of a function f (which is continuous and has two or more continuous derivatives), inflection points correspond to zeros of f'' (where its sign changes).

A **biflecnodal point**, or **biflecnode / fleflecnode** is a double point that's also an inflection point for both lines. they occur in quartics and above.

e.g.

$10y^4 - \frac{1}{5}x^4 + 2xy(3x^2y^2 - 1) = 0$

A **double point** is a point passed through twice on a plane curve (you can't do this for a function!)

Here, the point $(0,0)$ is passed through twice as we go along the curve $y^2 = x^3 + 2x^2$.

(I guess we want the solutions when $x=0$, and $y^2=0$ should have 2 solutions but in this case, they're $y = \pm 0 = 0$?)

An aside: It is important to distinguish between graphs of functions and plane curves - both can be written as equations in x and y, but a function needs a unique y for each x, while a plane curve is just the locus of all pairs (x,y) that satisfy the equation.

e.g. $y = x^2$

$x^2 + y^2 = 1$

The 'vertical line test', in which any vertical line must cross the curve once only, tells you if it's a function.

ORTHIC TRIANGLE

A 10-minute maths zine for #mathober 2026

by @stecks

An altitude is a line from one corner of a triangle, which hits the opposite side at 90°.

The incircle is a circle which lies tangent to all three sides of the triangle.

Given a triangle ABC there are various lines and shapes we can construct to find out more about it (and for fun).

If we bisect the angles at each corner of the triangle, these lines will hit the opposite edges of the triangle at three edge midpoints.

The medial triangle is made by joining the midpoints of the edges.

The orthic triangle (or altitude triangle) is made by joining the endpoints of the three altitudes.

The mid-arc triangle has its vertices where the angle bisectors meet the incircle (closest to the vertex you bisected at)

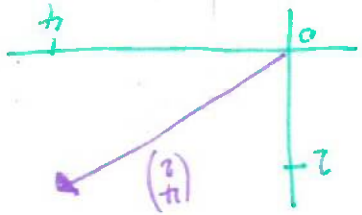
I have made a Geogebra file with all of these constructions so you can have a play!

katiesteckles.co.uk/mo19-ggb

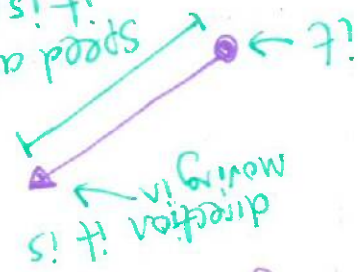
I ▽ triangles

A vector is (roughly) a list of numbers. We can think of a vector of length n as specifying an arrow in n-dimensional space.

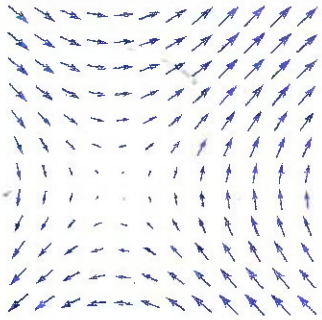
e.g. $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ in 2D



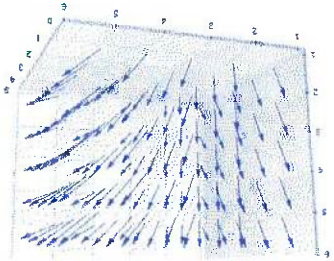
A vector has both a direction and a length, which makes them useful for modelling things like motion.



To visualise a vector field, we will often just show the vectors starting at integer coordinates (but it can be done anywhere)



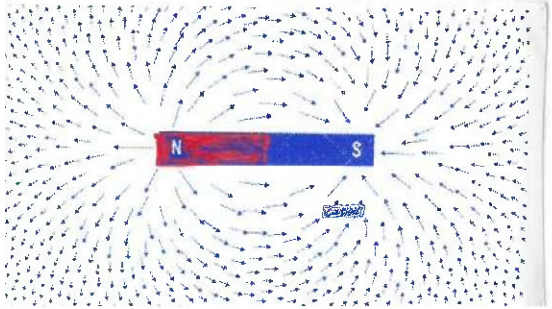
e.g. in 2D



in 3D

A vector field is a region of n-dimensional space with a vector attached at every point.

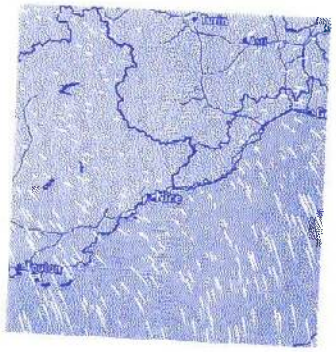
We can use a vector field to describe movements or other vector quantities, over a whole area at once - eg. a magnetic field:



The vectors point along (and in the direction of) the field lines at each point in space.

Because they're vectors we can do a lot of the same maths on them we could on a single vector, as well as describing overall properties of the field!

This vector field shows the wind speed over the south of France:



Yes that's right, it's a NICE VECTOR FIELD (sorry)

NICE VECTOR FIELD

A 10-minute Maths zine for #Mathober 2026

by @stecks

PRIME RING

A 10-minute Maths Zine for #mathober 2026

by @stecks

The prime ring is the subring generated by repeatedly adding 1.

- In $\mathbb{Z}_4$, it's the whole ring.
- In the ring of polynomials with coefficients in $\mathbb{Z}_4$, it's a copy of $\mathbb{Z}_4$ made up of the constant polynomials 0, 1, 2 and 3 (defined as $a + 0x + 0x^2 + 0x^3 \ldots$ $a \in 0, 1, 2, 3$)
- For an infinite ring, the prime ring is $\mathbb{Z}$.

The real numbers have some really useful properties for doing arithmetic.

If x, y, z are numbers, then

$x+y$ is also a number

$x \times y$ is also a number

$(x+y)+z = x+(y+z)$

$(x+y) \times z = x \times z + y \times z$

etc.

We also know that $-x$ exists, such that

$x + (-x) = 0$

and $\frac{1}{x}$ exists, such that

$x \times \left(\frac{1}{x}\right) = 1$

But we can also construct finite sets, with their own defined operations for $+$ and $\times$, which still behave this same way.

If we use only the numbers 1-5, with $+$ and $\times$ defined modulo 5 (so, $6=1$, $7=2$, and $5=0$) we call this $\mathbb{Z}_5$ and all the same things still work:

$3 + 4 = 7 \equiv 2$

$2 \times 3 = 6 \equiv 1$

In fact, we can display all this in tables for addition & multiplication:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

But we can also construct sets which only partly work: if we take $1-4$ with $+, \times$ Modulo 4 (called $\mathbb{Z}_4$) we no longer have $x \to \frac{1}{x}$ and we also sometimes get weird results like $x \times y = 0$, $x, y \neq 0$.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

no inverse so $\nexists 2^{-1}$

This kind of structure is called a RING, and it's still useful even if it's a bit broken. We can also define fun structures based on "rings", like "all polynomials with coefficients in $\mathbb{Z}_4$" e.g. $a + bx + cx^2 + dx^3 + \ldots$, $a, b, c, d \in \mathbb{Z}_4$

POLY-LOGARITHM

A 10-minute maths zine for #mathober2026

by @stecks

A polylogarithm is a function on the complex numbers, defined as (for $|z|<1$):

$$\mathrm{Li}_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s} = z + \frac{z^2}{2^s} + \frac{z^3}{3^s} + \frac{z^4}{4^s} + \cdots$$



Here, z is the input to the function, which can be any complex number whose modulus is <1, and s is a parameter: so this actually defines a whole family of functions!

We could also fix the value of z: when $z=1$ the polylogarithm becomes

$$\mathrm{Li}_s(1) = \zeta(s)$$

(sorry can't draw it)

the Riemann Zeta function!

$(\mathrm{Re}(s)>1)$

Surprise!



The reason it's called a polylogarithm is because when $s=1$, we have

$$\mathrm{Li}_1(z) = \sum_{k=1}^{\infty} \frac{z^k}{k} = -\ln(1-z)$$

To find other functions in the family, we input other values of s:

$\mathrm{Li}_2(s)$ is called the dilogarithm

$\mathrm{Li}_3(s)$ is called the trilogarithm

In general,

$$\mathrm{Li}_{s+1}(z) = \int_0^z \frac{\mathrm{Li}_s(t)}{t}\,dt$$

For non-positive integers, the functions are actually reasonably well-behaved, rational functions in z.



$$\mathrm{Li}_0(z) = \frac{z}{1-z}$$

$$\mathrm{Li}_{(-1)}(z) = \frac{z}{(1-z)^2}$$

$$\mathrm{Li}_{(-2)}(z) = \frac{z(1+z)}{(1-z)^3}$$

$$\mathrm{Li}_{(-3)}(z) = \frac{z(1+4z+z^2)}{(1-z)^4}$$



and $\mathrm{Li}_{(-4)}(z) = \dfrac{z(1+z)(1+10z+z^2)}{(1-z)^5}$

Generally, $\mathrm{Li}_{(-n)}(z) = \left(z\dfrac{\partial}{\partial z}\right)^n \dfrac{z}{1-z}$

DEVIL ON TWO STICKS

A 10-minute Maths zine for #mathober 2026

by @stacks

I have of course made a Geogebra file:

katiesteckles.co.uk/Mo23-ggb



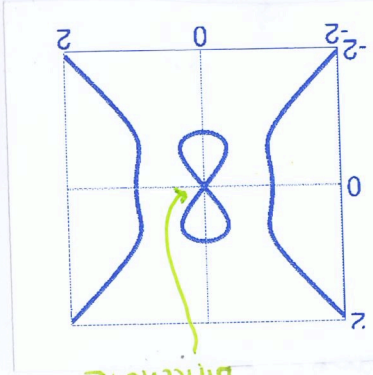
The "devil on two sticks" is a parameterised family of plane curves, defined in terms of two parameters a and b. It is given by

$$y^4 - b^2y^2 = x^4 - a^2x^2$$

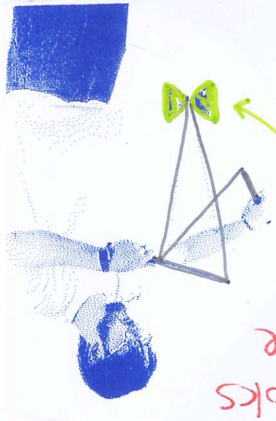
It was discovered by Gabriel Cramer, who I mainly already know of from Cramer's Rule in linear algebra, and also his badass portrait with "GAB. CRAMER" in the corner.



The Devil on two Sticks, also called the Devil's Curve, has a bitecnode* in it (for some values of a and b)



It's called the devil on two sticks because the figure -a-8 in the middle looks a bit like a DIABOLO (fun juggling doodad)

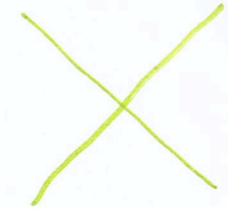


* see #mathober 2026 day 18

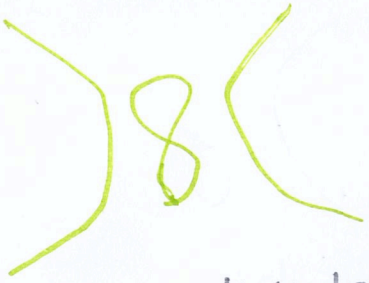
Depending on the values of a and b, you get different shapes:



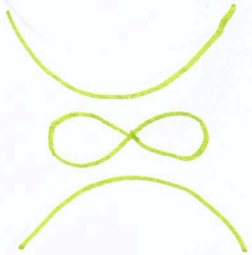
a=b=0



0≠a=b≠0



|a|>|b|



|a|<|b|

MULTI-PERFECT NUMBER

A 10-minute Maths zine for #mathober 2026

by @stecks

A perfect number is a number for which the sum of its proper factors (numbers < n which n is divisible by) is equal to the number itself.

e.g

$6 = 1 + 2 + 3$

and 6 can be divided by 1, 2 and 3

There are many known examples of perfect numbers:

$28 = 1 + 2 + 4 + 7 + 14$

$496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248$

$8128 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064$. Perfect!

There are (at time of writing) only 52 known perfect numbers, and it's unknown if there are infinitely many, or if any odd perfect numbers exist. So, if you're looking for more numerical perfection, how about you try--

MULTI PERFECT NUMBERS!

These are numbers whose factors add up to a multiple of the number:

$\sigma(n) = kn$

(σ = divisor function (sum of positive divisors), k = perfect number, n = a multiple of n)

Since the definition of a multiperfect number includes all factors (not just proper factors) all perfect numbers count as 2-perfect numbers.

e.g

$1 + 2 + 3 + 6 = 2 \times 6$

(12 = 12)

I made this diagram!

The smallest 3-perfect number is 120, as

$1 + 2 + 3 + 4 + 5 + 6 + 8 + 10 + 12 + 15 + 20 + 24 + 30 + 40 + 60 + 120 = 360 = 3 \times 120$

(I tried to make the same diagram for this but it's so long it came out really small)

CROSSED PRISM GRAPH

a 10-minute Maths zine for #Mathober 2026

by @stecks

A crossed prism graph is a graph made by following these steps:

1. Choose an even integer value for n e.g $n=4$

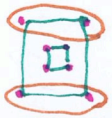
2 Draw a copy of the cyclic graph C_n (a loop with n nodes joined by edges in a cycle)



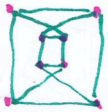
3. Draw another copy of C_n inside the first (in the same orientation)



4. Group the nodes on the outside ring into $n/2$ adjacent pairs

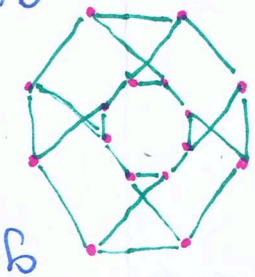


5. For each pair, draw two crossing edges to join the outside path to the corresponding pair on the inside, with a twist!



This can be done for any even n (but not odd n, as you can't do step 4). The resulting graphs all share some properties.

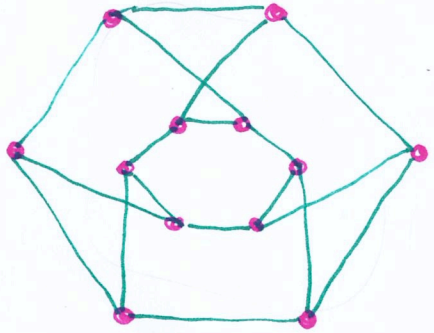
- They are cubic graphs - every node has 3 edges meeting at it
- They are bipartite If you 2-colour the graph, so no pair of nodes share colour one joined, the nodes split into two groups.



- They are vertex transitive Each vertex is joined to the graph in the same way so you could pick two and rearrange the whole graph so one becomes the other

The $n=6$ case is called the Franklin Graph

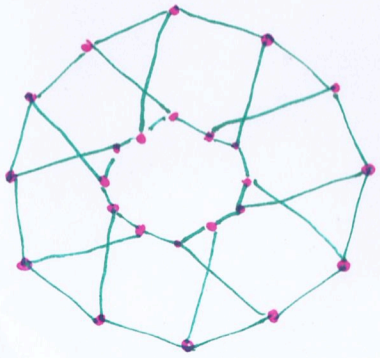
It shares all these properties, but also has a cool story.



It can be drawn on the surface of a Klein bottle so the result requires 6 colours to colour in - and was the basis of the proof that 6 colours can be needed!



$n=12$ case: Ct42 in the book An Atlas of Graphs.



I have made a Geogebra so you can try dragging the nodes around:

Katiesteckles.co.uk/M025-ggb



UNIMODULAR LATTICE

A 10-Minute Maths zine for #Mathober 2026

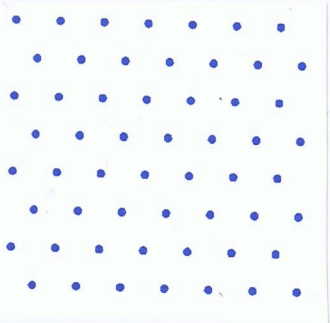
by @stecks

In maths, we use the term LATTICE to describe pretty much what you would imagine: a spaced-out grid of points in n-dimensional space.

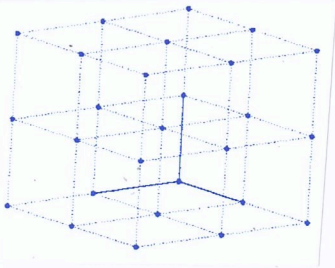
e.g. A 1D lattice:



A 2D lattice:



A 3D lattice:

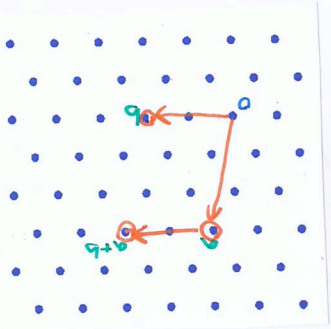


But it's even better than that! We can consider the lattice to be a group structure: so if we take two points on the lattice (relative to 0) we can "add" them together and get another point on the lattice.

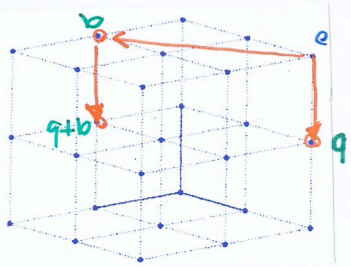


$-\pi + 2\pi = \pi$

In 2D:



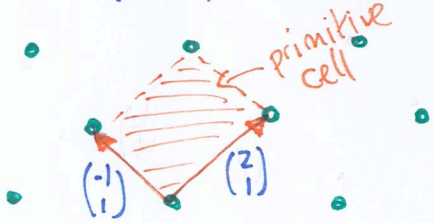
In 3D:



A UNIMODULAR LATTICE is one whose primitive cell has volume 1. Examples include:

- $\mathbb{Z}^n$ (yawn)
- E_8 (a lattice in 8D space consisting of vectors of length $\sqrt{2}$)
- The Leech Lattice, a 24-dimensional lattice where each unit ball at a lattice point is tangent to 196,560 neighbours!

If we write down the vectors that point to the corners of the primitive cell as a matrix M, det(M) is the volume of the primitive cell.



$\det\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = 3$ ∴ area of primitive cell = 3

We also require that any two points in the lattice are distinct - there's a minimum distance between them - and that the lattice covers the whole space.

The FUNDAMENTAL DOMAIN or PRIMITIVE CELL of a lattice is the individual tile between a set of lattice points.

INDEFINITE QUADRATIC

A 10-minute Maths zine for #mathober 2026

by @stecks

A polynomial is an expression made by combining powers of a variable x with (usually) coefficients in $\mathbb{R}$.

Generally,

$a+bx^2+cx^3+dx^4+\dots$

e.g. $2x+3$ $\quad x^2+2x+1$

x^3+4x^2+5

We classify polynomials based on the highest power of the variable present:

x^0 : constant

x^1 : linear

x^2 : quadratic

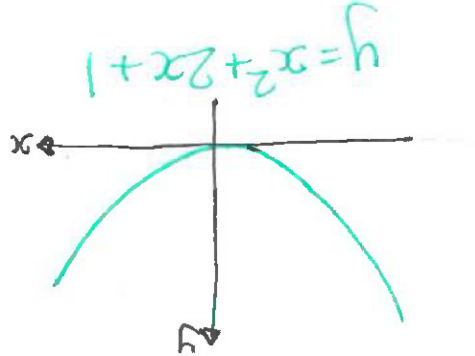
x^3 : cubic

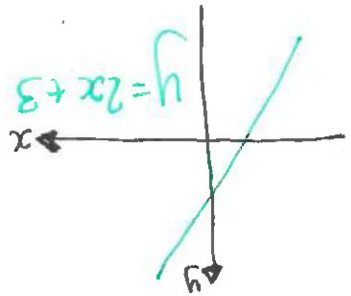
x^4 : quartic

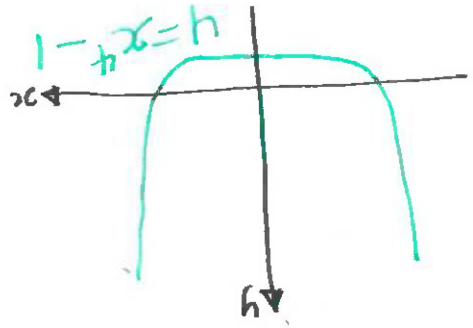
x^5 : quintic

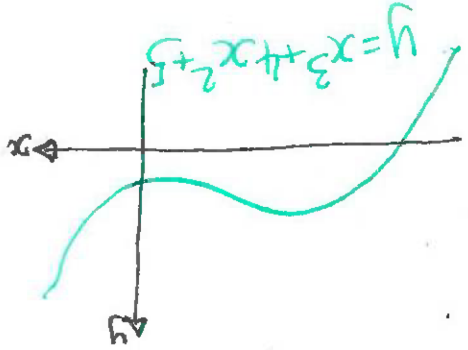
x^6 : sextic

If we set a polynomial in x equal to y we can plot it:

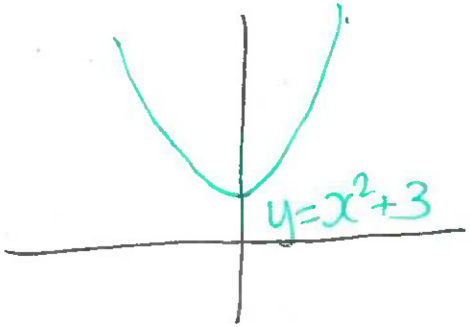








If the highest power of x is even, as in a quadratic or quartic, it may be the case that the plot of the function doesn't cross the x-axis.



This means the polynomial has no real roots

For a quadratic, we can use the discriminant to check.

ax^2+bx+c ← quadratic

b^2-4ac ← discriminant

(yes, it's the bit in a $\sqrt{\cdot}$ from the quadratic formula:)

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

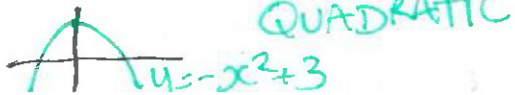
If $b^2-4ac<0$ there are no real roots + we say the quadratic is (positive or negative) definite

↑ (depending on the sign of x^2)

If $b^2-4ac=0$ there's 1 root

If $b^2-4ac>0$ there are two real roots + the curve goes above + below the x-axis

It is an INDEFINITE QUADRATIC



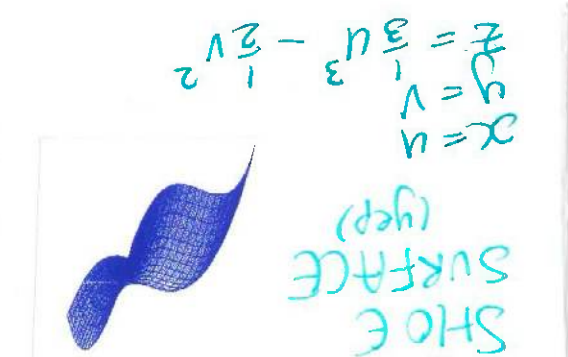
CONIC DOUBLE POINT

A 10-minute Maths zine for #mathober 2026

by @stecks

A cubic surface is a set of points in 3D satisfying a cubic equation in x, y and z.

There are many famous examples, and for one I don't have to make up funny names for any of them as they are already widely winning on that front.



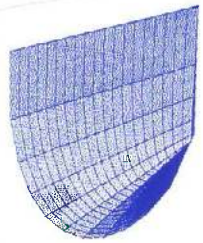
$$x_0+x_1+x_2+x_3+x_4=0$$
$$x_0^3+x_1^3+x_2^3+x_3^3+x_4^3=0$$

CLEBSCH DIAGONAL CUBIC



$$4(x^3+y^3+z^3+w^3)-(x+y+z+w)^3=0$$

THE CAYLEY CUBIC



$$x=v\cos u$$
$$y=v\sin u$$
$$z=\sqrt{a^2-b^2\cos^2 u}$$

WALLIS' CONICAL EDGE

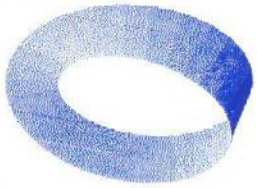
$$x=u$$
$$y=v$$
$$z=\frac{1}{3}u^3-\frac{1}{2}v^2$$

SHOE SURFACE (yep)

$$\frac{DR}{-R^2y+x^2y+y^3-2Rxz-2xyz-2y^2z+yz^2=0}$$

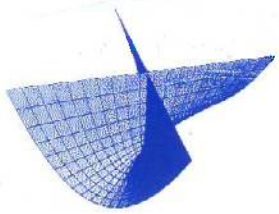
$$x=(R+s\cos(\tfrac{1}{2}t))\cos t$$
$$y=(R+s\cos(\tfrac{1}{2}t))\sin t$$
$$z=s\sin(\tfrac{1}{2}t)$$

actually a cubic surface!



THE MÖBIUS STRIP is also

These surfaces often have singularities (similar to double points in 2D curves) where the surface intersects itself.

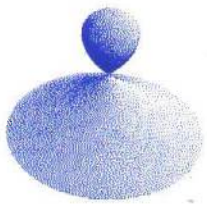


THE WHITNEY UMBRELLA (yep)

$$x=uv,\ y=u,\ z=v^2$$

Now we're in 3D, this can happen in a few different ways. One is a CONIC DOUBLE POINT, denoted A_1, as seen in

THE DING-DONG SURFACE (seriously)



$$x^2+y^2=(1-z)z^2$$

Other kinds of singularities include



uniplanar and biplanar double points ($A_{2\text{-}5}$, $D_{4\text{-}5}$) which are more like cusps (where the surface is pinched together) and also elliptic cone points ($\tilde{E}_6$) which come in many variations.





If instead we extend the edges of both triangles and their pairwise intersection points are collinear, we say the triangles are perspective from a line.

If two triangles have the property that the lines joining their corresponding corners all meet at a point, we say they are perspective from a point.

TWO TRIANGLE THEOREM

Desargues' TWO TRIANGLE THEOREM says that:

If two triangles are perspective from a point ⟹ THEN ⟹ They are also perspective from a line !

This theorem is self-dual: if you swap the words "point" and "line" in its definition, you get the same theorem back.

(All theorems in projective geometry have a dual statement obtained in this way.)

Other things that are also self-dual:

the tetrahedron (as actual solid)

this graph (as a dual graph)

a Hilbert space (as a normed space)

TWO TRIANGLE THEOREM

A 10-minute Maths zine for #Mathober 2026

by @stecks

SPHEROIDAL WAVE FUNCTION

A 10-minute maths zine for #mathober 2026

by @stecks

The wave equation is a second order partial differential equation used in modelling waves. It can be written as:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \frac{\partial^2 u}{\partial z^2} \right)$$

in 3D (x, y, z) - coordinates where u is a field at scalars, depend at each point, and t is time.

It can be used to model waves, or standing wave fields, such as waters, sound or seismic waves or electromagnetic waves.

It is often useful to study how these functions behave in a finite situation, such as a membrane where the edges are clamped in place so they can't move.

Equi-ateral triangle: 1852 (Lamé)

Rectangular membrane: solved 1829 (Poisson)

Circular Membrane solved 1862 (Clebsch)

of the Clebsch Surface then #mathober 28th!

The Helmholz equation used in solving the wave eqn)

Another way to constrain the system is to place it on the surface of a sphere - where we can model systems on the surface of the earth like weather systems, as well as electromagnetic fields, gravitational fields, and planetary bodies and stars, as well as in 3D graphics (eg for lighting effects)

In order to solve the equations (such as the Helmholz equation) we write them in spherical coordinates.

Then we solve using the technique of separation of variables, rewriting the equation so each variable is on a different side of the equation.

This is possible for the wave equation, but only in spherical coordinates (I think? This is not really my area but it is all very cool).

EXISTENTIAL CLOSURE

(or, CLOSURE)

A 10-minute Maths zine for #Mathober 2026

by @steclcs

In this zine, I will aim to explain the concept of the EXISTENTIAL QUANTIFIER so well that you will experience an emotion known as EXISTENTIAL CLOSURE.

In predicate logic, we can define a predicate $P(x)$ which takes an input x from a set X. It can be any statement about x which has a defined true or false value for a given x.

For example, if $X = \mathbb{Z}_+$, we could define

$P(x)$ = "x is prime"

and

$Q(x)$ = "x is even"

(whether these are true or false depends on what the value of x is)

The Existential Quantifier, denoted $\exists$ is used to mean that an instance of something does exist. We say

$\exists x \in X\, P(x)$ to mean "there exists an x in X for which x is prime"

(assuming the same definition of $P(x)$ as before).

$\exists x \in X\, P(x) \wedge Q(x)$ means "there exists an x in X such that x is prime and x is even"

$\exists x \in X\, \neg P(x)$ means "there exists an x in X such that x is not prime"

(a truly mind-blowing statement in maths - it only we could prove it)

If the set X is instead $X = \{x \in \mathbb{Z}_+ \mid x > 3\}$

(the whole numbers that are greater than 3)

we can say

$\neg\exists x \in X\, P(x) \wedge Q(x)$

ie "there does not exist x in X such that x is prime and x is even"

Are you feeling existential closure yet?

We also use the symbol $\nexists$ (the negation of the existential quantifier) to mean "there does not exist" or alternatively "for no" as in, "for no x in X does this predicate hold"

The symbol for the existential quantifier is sometimes called a TURNED E.

Unicode U+2203 LaTeX \exists

It was introduced in 1896 by Giuseppe Peano, and popularised by Bertrand Russell! $\exists\heartsuit$